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A Fuzzy Expert System for Solving Possibilistic Multiobjective Programming Problems

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This article proposes a fuzzy expert system (FES) for solving the multiobjective possibility programming problems. This FES uses decision-making rules to obtain a solution under qualitative information conditions. To solve such a problem, two approaches are proposed for estimating the solution-rules matrix. A fuzzy inference algorithm is developed. Architecture of the FES is presented. Potential possibilities of the expert system are showed on model examples.

Keywords: Multiobjective decision making, Fuzzy variable, Linguistic variable, Fuzzy Measure, Fuzzy Inference, Linguistic approximation, Fuzzy expert system.

1. Introduction.

2. Problem Formulation

The purpose of this paper is to solve the following multiobjective linear programming problem involving fuzzy linguistic (possibility) parameters.

$$
\begin{aligned}
&\max\ f_k(c^k, x), k = 1,\dots,l;\\
&s.t\\
&\sigma\{\sum_{j=1}^{n} a_{ij}x_j = b_i(\gamma)\} \geq \tau_i, i = 1,\dots,n, \qquad (1)\\
&x \in X, x_j \geq 0;
\end{aligned}
$$

Where $c^k = (c_{k1}, \dots, c_{lj})^T$, x is an n-dimensional vector i.e. $x \in E^n$, where E^n is measurable *n*-measured Euclidean space, $x \subset E^n$ is a set, satisfying restrictions of a special type $a_{ij} \in E^1$, $b_i(\Upsilon)$ are fuzzy (possibility) variables, defined in a possibility space (Γ, P(Γ), σ). In this model of fuzziness [3], $\Upsilon \in \Gamma$ is a set of elements, P(Γ) is the set of all subsets of Γ, a is a fuzzy measure, satisfying the following conditions:

1) $\sigma(\phi) = 0$, $\sigma(\Gamma) = l$;

2) $\sigma(U_\propto A_\propto = \sup \sigma(A_\propto)$, $\forall A_\propto \in P(\Gamma)$.

Naturally, as in probability theory, the possibility space (Γ, P(Γ), σ) is in the background, and distribution functions of possible values for fuzzy parameters of b_i come forward. The function μ_{bi} (t) is the possibility that fuzzy variable b_i can take on value *t* and be determined as follows:

$\mu_{bi}(t) = \sigma\{\gamma \in \Gamma \mid b_i(\Upsilon) = t\}, \forall \in E^1$

Model (1) can be interpreted as follows: Let the vector $b(\Upsilon) = (b_i(\Upsilon), \ldots, b_m(\Upsilon))$ contain possible values of resources (raw products, materials, …ect.), vector $x = (x_1, \ldots, x_n)$ be a vector of manufactured volume, a_{ij} be an expense volume of resource *i* for product *j*, matrix $c\ (c^1, \ldots, c^l)^T$ be the price, cost, ..ect vectors of objective functions. It is required to find an output production plan $x = (x_1, \ldots, x_n)$ providing the maximum profit, minimum cost, and…, ect with the possibility to satisfy a resources expense balance, If the quantity of these resources is not less than threshold values $\tau_i \in (0, l]$, $i = l, \ldots, m$.

3. Yazenin's Method

Yazenin [1] has obtained a solution of linear programming problem (*L1*) with fuzzy parameters in the constraints where the right hand sides of the constraints contain possible values of resources. The solution processes based on using solving rules (5) to convert the linear programming problem *L1* into its equivalent *L2*; where D = $\{d_{ij}\}$ is an $n * m$ solving rule matrix deterministic coefficients $d_{ij} \geq 0$.

$$Ll : \max f\ (c\ , x), k = 1, \ldots, l; \tag{2}$$

s.t

$$\sigma\{\sum_{j=1}^{n} a_{ij} x_j = b_i(\gamma)\} \geq \tau_i, i = 1, \ldots, n, \tag{3}$$

$$x \in X, x_j \geq 0;$$

$$L2 : \max \eta$$

s.t

$$r_0 \leq (\eta - (c, D\alpha)) / (c, D\beta) \leq R_0, \tag{4}$$

$$r_i \leq \alpha_i - (a^i, D\alpha)) / (\beta_i + (a^i, D\beta) \leq R_i, i = 1, \ldots, m,$$

$$d_{ij} \geq 0, i = 1, \ldots, n, j = 1, \ldots, m, k = 1, \ldots, l$$

where $a^i = (a_{i1}, \ldots, a_{in})$, $R_i = (1 - \tau_i)/2$, $r_i = (\tau_i - 1)/2$

$$x(\Upsilon) = D\, b(\Upsilon) \tag{5}$$

The functional dependence (5) gives a method for correct plan x with the given information about resources, that does not require to solve (2) and (3). The matrix of the rule (5) can be estimated according to a priori information about the distribution of possible values of vector $b(\gamma)$. The fuzzy variables used in this model are the normal and symmetric triangular fuzzy variables. Where ;

$$\mu_{b_i}(y) = \exp\{-((y-a_i)/\beta_i)^2\} \in N(\alpha_i, \beta_i), y \in E^1 \tag{9}$$

$$\mu_{b_i}(y) = \exp\{0, \min\{1-c_{b_i}(y-a_i), 1+c_{b_i}(y-a_i)\}\} \in Tr(\alpha_i, \beta_i), y \in E^1 \tag{10}$$

4. Problem Solving

Model *L2* can be represented by model *M1* by replacing constraint (4) with constraints (11) and (12). P3 can be represented by two constraints (2) and (3) where $f_{k\alpha}(d_{ij}) = (c^k, D\alpha) + r_o(c^k, D\beta)$, and $f_{k\beta}(d_{ij}) = (c^k, D\alpha) + R_o(c^k, D\beta)$

$$f_{k\alpha}(d_{ij}) \le \eta, \tag{11}$$

$$f_{k\beta}(d_{ij}) \ge \eta, \; k = 1, \ldots, l, \tag{12}$$

So, P3 can take the following form

$$\begin{aligned}
\text{L3}: \ & \max \eta \\
& s.t. \\
& f_{k\beta}(d_{ij}) - \eta \ge 0, \\
& f_{k\alpha}(d_{ij}) - \eta \le 0, \\
& r_i \le \alpha_i - (a^i, D\alpha))/(\beta_i + (a^i, D\beta) \le R_i, i = 1, \ldots, m, \\
& d_{ij} \ge 0, i = 1, \ldots, n, \; j = 1, \ldots, m, k = 1, \ldots, l
\end{aligned}$$

For solving the above mentioned problem (1), we will solve model *L3* for each objective function $f_k(c^k, x)$ to construct the membership function (13) and (14) for $f_{k\alpha}(d_{ij})$ and $f_{k\beta}(d_{ij})$ respectively. Afterwards, based on the Zimmermman approach for solving multiobjective linear programming problems, we will solve *M1* to find the best compromise solution.

$$\mu_k\left(f_{k\alpha}(d_{ij})\right) = \begin{cases} 1 & f_{k\alpha}(d_{ij}) \ge U_{k\alpha} \\ \dfrac{f_{k\alpha}(d_{ij}) - L_{k\alpha}}{U_{k\alpha} - L_{k\alpha}} & L_{k\alpha} \le f_{k\alpha}(d_{ij}) \le U_{k\alpha} \\ 0 & f_{k\alpha}(d_{ij}) \le L_{k\alpha} \end{cases} \tag{13}$$

$$\mu_k\left(f_{k\beta}(d_{ij})\right)=\begin{cases} 0 & f_{k\beta}(d_{ij})\geq U_{k\beta} \\ \dfrac{U_{k\beta}-f_{k\beta}(d_{ij})}{U_{k\beta}-L_{k\beta}} & L_{k\beta}\leq f_{k\beta}(d_{ij})\leq U_{k\beta} \\ 1 & f_{k\beta}(d_{ij})\leq L_{k\beta} \end{cases} \qquad (14)$$

$$\begin{aligned} &\text{M1: } \max \lambda \\ &\quad s.t \\ &\quad f_{k\alpha}(d_{ij})-(U_{k\alpha}-L_{k\alpha})\lambda \geq L_{k\alpha}, \\ &\quad f_{k\beta}(d_{ij})+(U_{k\beta}-L_{k\beta})\lambda \leq U_{k\beta\alpha}, \\ &\quad r_i \leq \alpha_i-(a^i, D\alpha))/(\beta_i+(a^i, D\beta)\leq R_i, i=1,...,m, \\ &\quad d_{ij}\geq 0, i=1,...,n,\ j=1,...,m, k=1,...,l \end{aligned}$$

Conclusion

In this paper we presented architecture of FES for solving MODM problem with fuzzy (possibilistic) parameters in the right hand side. The general approach for solving such a problem has been introduced combined with an illustrative example. The advantages of this approach are it's not necessary to resolve the problem each time the DM changes his preferences; i.e. this method gives the DM an on-line answer, and let the DM to reflect his/her preference in linguistic form.

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