Scheduling a Precision Forge in a Lean Manufacturing Environment*

Chinthaka Hettitantri, Xin Huang, Xijin Yan

Abstract: Jet Engineering, Inc., a company based in Lansing, MI specializes in precision forging, machining, and full finishing capabilities for the orthopedic and aerospace industries. Currently in this company, the operations of each precision forge process are controlled by a single operator. The efficiency of the operator determines the processing time needed for each product.

In this work, the precision forging of an orthopedic implant was analyzed in order to minimize the processing time per part in a steady-state condition while maintaining the temperature history requirements. To extend the application of our model to other products, we developed a lean schedule as a recommendation for forging processes. Following this recommendation, any operator should be able to produce a given number of parts in the least amount of time. The operator will then be able to develop similar lean schedules for other products regardless of his/her experience.
2. Introduction

2.1 Physical Model
Jet Engineering, Inc., Lansing, MI is one of the few companies in the world capable of producing precision, near-net shape forgings (near final part dimensions) from what have come to be known as the "super-alloys" (titanium, cobalt-chrome) and other non-ferrous metals, as well as a number of types of stainless steel. It manufactures a wide variety of products required for orthopedic implants, aerospace, automotive, and sports equipment.

The focus of this project concerns one specific product: an orthopedic implant. An orthopedic implant represents one of Jet Engineering’s basic designs and was chosen for the sake of simplicity and concreteness. There are many processes that take place in the production of a typical orthopedic implant. A billet of super-alloy is cut into a shape (cylindrical, prismatic, or other simple solid), precision forged, finished, and packaged. We are primarily concerned with the precision forging process, which consists of heating and forging the billet (this process may also involve a subsequent heating and forging operation depending on the part).
Figure 1. The forging station

As shown in Figure 1, each forging station consists of a forging station, squashing station and (usually) four furnaces (which heat and soak the billets prior to forging). The station is controlled by a single operator.

The entire forging process of a single billet consists of four steps:

1. **The Preheating Step.** In this step, the billet is inserted into a furnace and heated to its forging temperature.

2. **The Squashing Step.** After the billet reaches its forging temperature, it is removed from the furnace and taken to station 2, where it is squashed into a pre-designed shape determined by the requirements of the different structures. The squashed billet is then set back into a furnace (it can be the furnace used for preheating or another) to regain its forging temperature.

3. **The Forging Step.** Once the squashed billet regains its forging temperature in the furnace, it can then be forged into its final shape at station 1. And then it is removed to station 2.

4. **The Finishing Step.** Finally, at station 2, the forged billet will be trimmed and then adjusted into its final shape.

In order to meet product quality standards, the billet needs to be heated to its forging temperature and then "soaked" at that temperature for the proper amount of time, neither too long, nor too short. That is the meaning of “temperature history,” which is the most difficult and important part in the precision forging process.

Until now, the operator intuitively determines how many billets are placed into a furnace, how long they are heated, and how the above four steps are coordinated with each other. Different operators have different schedules; therefore, the operation efficiencies vary.

### 2.2 Project Objective

The objective of this project is to analyze and optimize the detailed operation of the forge station in the context of a lean manufacturing environment (see Appendix E). We try to develop a lean scheduling tool for a station processes so as to ensure that the lot in question would be produced in the least amount of time, while also ensuring that the part could be forged with the proper temperature history and maximum equipment usage. Moreover, based on the model, the optimized scheduling tool was designed to be adaptive to the variables, such as the projected production type, the number of furnaces, number of operators, number of billets put in a furnace, the product size and thus the corresponding placement, etc. The model is also intended to be useful for Jet engineering personnel to develop similar lean schedules for other parts.
3. Analysis
3.1 Site Visiting and Data Analysis of the Current Model

What follows is a simulation of the procedures performed by a single operator recorded during a site visit. An analysis of the steps of the forging process helped to determine the time factors involved for each step of the process. The time factors, based on the forging process for an orthopedic implant described in the introduction, are as follows.

1. The Preheating Step. The operator starts with pretreating the stations, while loading the raw billets at the same time. Here, 36 pieces of raw billets are divided into three groups, 12 for each and then loaded into three furnaces as shown in Figure 2, under a preset temperature (this supposes that the furnaces are already at the preset temperature). Then, he waits for the amount of time required for the billets to reach their forging temperatures. The preheating time is denoted by $t_p$ and for this forging process, $t_p\approx 15$min.

![Figure 2. Current placement of the preheating billets in the furnaces](image)

2. The Squashing Step. After $t_p$ (15 minutes), (after all of the billets have reached their forging temperature) the operator begins to remove the billets, one by one, to the squashing station, strikes them into an intermediate shape and then inserts them into the empty furnace as shown in Figure 3. The symbol $t_s$ denotes the operating time involved in squashing a single billet. For billet i, it has to stay in the furnace until the previous i-1 items are completed, namely, for the time $t_p+(i-1)t_s$ (preheating time plus the total operating time for the previous i-1 billets). Once it is squashed, it will stay in the furnace for the time $(36-i)t_s$ until all the following items are finished. See billet 5. It has to stay in the furnace for the time $t_p+4t_s$ before being squashed and $31t_s$ after being squashed until all of the remaining 31 billets get squashed. Overall, supposing the operator works continuously and also ignoring the possibilities of
a machine breaking down, it will take him a time 36t, to finish squashing all of the billets. See Table 2 in Appendix A for the detailed list. For this process, t~10 seconds.

![Furnace Diagram]

Figure 3. Current placement of the reheating billets in the furnaces

3. The Forging Step. After squashing all of the raw billets, the operator begins the forging process for each one. In this Forging process, the operator follows the following steps: pretreating the forge die(s) with lubricant, loading a heated billet, striking the billet one or more times, and extracting the forged part. The forged billets are placed in a container next to the trimming/squashing station. The forging time for a single billet is denoted by tf, and tf~25s. Therefore the total time required to finish all of the billets is 36tf. See Table 2 in Appendix A.

4. The Reheating Step. The reheating step occurs simultaneously with both the squashing and the forging steps. Its purpose is to make up for the temperature lost during the squashing operation, i.e., to let the billets regain their forging temperatures. Consider the k-th billet, before it is forged, it has to stay in the furnace for the time (36-k)tf + (k-1)tf, where (36-k)tf is the time the billet stays in the furnace after it was squashed until all the following billets in the same cycle get squashed. If tf is the forging time for one billet, then (k-1)tf is the time the billet must stay in the furnace during the forging cycle before it is forged. See Table 2 in Appendix A for the detailed list.

5. The Finishing Step. Finally, the operator trims the billets placed in the container, checks the quality of manufacturing to satisfy production standards, and puts them in another container. If tt is the trimming time for one single billet, the total operation time for the 36 billets will be 36tt. See Table 2 in Appendix A for this process.

In addition to the above analysis, there are 2 “problems” of importance.

1. First, we need to pay attention to the time a billet needs to regain its “lost temperature”. During this re-heating step (Step 3), we should consider the time, tr, which takes to make up for the heat lost of one billet when it is being squashed. If tr is less than or equal to 35tf (the time the first billet stay in the furnace after it was squashed), then it is ready to be forged and no waiting is needed. Otherwise,
the operator has to wait for the time difference \(t_r - 35t_s\). However, this problem is ignored in the real forging process.

2. Once the processing gets started, to save working time, the operator begins to load the three furnaces for another 36 raw billets. He then finishes trimming the first set of billets while the second new set preheats in the furnaces. So, it then follows that from the second cycle, the preheating time \(t_p\) can be reduced to \(t_p - 36t_s\).

Therefore, to simplify our model at the beginning, we suppose the operator works continuously and we ignore other minor unexpected time losses. For a production of 360 products, the total producing time \(t_a\) can be obtained by the following formula:

\[
t_a = 10 \sum_{i=1}^{5} t_i + 36t_l = 10[(t_p - 36t_s) + (36t_i) + (t_r - 35t_s) + (36t_l)] + (36t_l)
\]

\[
= 10[t_p + t_r + t_s + 36t_l] + (36t_l) .
\]

Applying this formula to the recorded data (\(t_s=10s, t_f=25s, t_i=10s, t_{p,max}= 25min, t_{p,min}=15min, t_{r,max} = 12min, t_{r,min} = 7min, M=4, N=360\)), the minimum total producing time is:

\[
t_a = 10*[15*60 + 10 + 7*60 + 36*25] + (36*10)
\]

\[
= 22660s (6h 17min 40s) .
\]

### 3.2 Analysis of Temperature History Requirements

In the preheating step, the metal is heated to its forging temperature and soaked at that temperature for a proper amount of time: neither too long, nor too short. For each type of billet, there exist two time limits, \(t_{p,max}\) and \(t_{p,min}\). Depending on the properties of different materials and shapes, \(t_{p,max}\) is the specific time limit past which the billets will overheat, leading to material waste. The time limit \(t_{p,min}\) is the minimum time for the billet to reach its forging temperature. Therefore, the total time for which the billets should stay in the furnace must be between \(t_{p,max}\) and \(t_{p,min}\). There is, however, an important difference between these two important temperatures. To a specific type of billet, \(t_{p,min}\) is a variable. Because the smaller the number of the billets in the furnace, the less the time it takes for all of the billets to reach their forging temperature, then the smaller the \(t_{p,min}\). Hence, it is possible to reduce the preheating time, as well as the overall amount of manufacturing time, by reducing the number of billets preheated in the furnace.
After the squashing step, the makeup time for the billet to return its forging temperature due to the temperature loss in this step should also satisfy the temperature history requirement. Thus, $t_{r,max}$ and $t_{r,min}$ come into play.

However, the current model does not strictly ensure the temperature history requirement. The last billet stays in the furnaces for about 17 minutes, which is longer than the upper limit, $t_{r,max} (12\text{ min})$. Overheating may have an influence on the quality of the final product in a negative way. To solve this problem, it is necessary to reduce the number of preheating billets in the forging cycle, which can be achieved by either reducing the number of furnaces at the forging station or reducing the number of preheating billets in one furnace. We chose the latter approach, because reducing the number of preheating billets in one furnace can reduce the minimum preheating time $t_{p,min}$ so as to reduce the total producing time.

### 3.3 Other Problems
The limited space in the furnace does not allow the operator to reach the billets in the rear part of the furnace when there are billets in front of them. If the first input billets are the last ones removed from the furnace, then they will be heated for an excessive amount of time. That is the reason for some billets to overheat.

### 4. Model Development
The requirements of Jet Engineering have necessitated the development of two different models. The first model employs one operator at one forging station, a fashion similar to the current situation at Jet Engineering. Since the operator cannot operate both the forging and the squashing stations simultaneously, there always exists one idle station. A second model employs two operators at one station to allow for the maximum use of all equipments. In each of these two models we will try to achieve two purposes. One is to satisfy our project requirements (to achieve a minimum time threshold and to maintain the temperature history of the billets). The other is to satisfy the steady-state requirement for the Jet Engineering Inc. The steady state requires that each time when the operator inserts a raw billet, he must have finished a final part. Further this process must happen continuously without interruption.

The first problem to be solved is the placement of billets in the furnaces. Since all of the billets in the steady-state process are treated in the same way, they are preheated and reheated for the same preheating time and reheating time. (The first input should be removed first.) So a new preheating billet cannot place in front of another preheating billet or a new reheating billet cannot place in front of another reheating billet. However, since the reheating time is shorter than the preheating time, the reheating billets stay in the furnaces a shorter duration than the preheating billets. It is thereby possible then, to insert a preheating billet first and then put a reheating billet in front of it. According to the size of furnaces in Jet Engineering Inc., the placement of the billets in furnaces should occur in the following way as shown in Figure 4.

![Diagram](image-url)
Figure 4. New placement of the billets in the furnace

Compared with the current placement (See Figure 2 and 3), the number of billets in one furnace is reduced. Thus, the minimum preheating time can be reduced to 12 minutes while the minimum reheating time can be reduced to 6 minutes.

According to the steady-state condition, during each step interval $t_0$, one raw billet should be put into the furnace, one preheated billet should be squashed, one squashed billet should be forged and one forged billet should be trimmed. For each time interval $t_0$, there must be one output. This process at the station does not change. If $N_p$ is the number of preheated billets and $N_r$ is the number of reheated billets in the furnace, $N_p$ and $N_r$ do not change either. The preheating time of any billet then will be about $t_p^\text{max} = (N_p-1)\cdot t_0$. Additionally, the reheating time of any billet will be about $t_r = (N_r-1)\cdot t_0$. To check the temperature history requirement, we developed two inequalities:

1. $t_r^\text{max} \geq (N_r-1)\cdot t_0 \geq t_r^\text{min}$, \hspace{1cm} (1)
2. $t_p^\text{max} \geq (N_p-1)\cdot t_0 \geq t_p^\text{min}$, \hspace{1cm} (2)

For this specific product, $t_p^\text{max} = 2\cdot t_r^\text{max}$. It follows then that $N_p = 2\cdot N_r = 2\cdot n$ could be a good solution. So the inequalities (1) and (2) become

1. $t_r^\text{max} \geq (n-1)\cdot t_0 \geq t_r^\text{min}$, \hspace{1cm} (3)
2. $t_p^\text{max} \geq (2\cdot n-1)\cdot t_0 \geq t_p^\text{min}$. \hspace{1cm} (4)

By substituting the time interval $t_0$ into the inequalities (3) and (4), we can find $n$.

If the number of raw billets in one furnace is $m$, we can calculate the number of furnaces $M$ needed in one forging station by

$M = 2\cdot n / m$.

The only difference between our two models is the step interval, $t_0$.

**4.1 One Operator Model**
Since the operator can only deal with one step at a time, he or she has to spend time $t_s$, $t_f$ and $t_t$ for squashing, forging and trimming in one operation time interval, $t_o$. And there might be a waiting time $t_{wf}$ before forging and $t_{ws}$ before squashing. The equation for the step interval, $t_o$, then is,

$$t_o = t_{ws} + t_s + t_{wf} + t_f + t_t .$$

The times $t_s$, $t_f$ and $t_t$ cannot be reduced because they depend on the efficiency of the operator. Therefore, to minimize the total operation time $t_o$, the operator will have to work all of the time. If there is no waiting time, the operation time $t_o$ can be reduced to

$$t_o = t_s + t_f + t_t .$$

Since $t_o$ is just the time the operator spends on one product, it is also considered the operation time of one billet.

The following equations check the temperature history requirement.

$$t_p = 2^n t_o - t_s ; \quad t_{p,min} \leq t_p \leq t_{p,max} ,$$

$$t_r = (n - 1)t_o ; \quad t_{r,min} \leq t_r \leq t_{r,max} .$$

Since $t_p \leq t_{p,max}$ and $t_r \leq t_{r,max}$, there is a breaking time range $t_{p,max}$, in which the operators can either deal with any unexpected troubles that occur at the station or they can rest without the risk of billet-overheating.

$$t_{bp} = t_{p,max} - t_p ,$$

$$t_{br} = t_{r,max} - t_r ,$$

$$t_{p, max} = \min(t_{bp}, t_{br}) .$$

Finally, the following equation is designed to calculate the time needed to finish $k$ billets

$$t(k) = t_o (3^n + k - 2) + (t_f + t_t) .$$

Substituting the total number of billets $N$ for $k$, the total operation time is:

$$t_o = t(N) = t_o (3^n + N - 2) + (t_f + t_t) .$$

**Application:** Applying the current data ($t_s = 10s$, $t_f = 25s$, $t_t = 10s$, $t_{p,max} = 25min$, $t_{p,min} = 12min$, $t_{r,max} = 12min$, $t_{r,min} = 6min$) into this model, then

$$t_o = t_s + t_f + t_t = 45s ,$$

$$12*60 \geq (n - 1)t_o \geq 6*60 .$$
25*60 ≥ (2*n - 1)*t₀ ≥ 12*60 .

Thus we got

17 ≥ n ≥ 9 .

The best choice is the minimum even number, eg. 10. The number of raw billets in the furnaces will be 20 (Nₚ=2*n); and the number of reheated billets will be 10 (Nᵣ=n).

Following this, all other parameters can be calculated accordingly:

\[
\begin{align*}
M &= \frac{2*n}{m} = \frac{2*10}{4} = 5 , \\
\text{The number of furnaces:} & \\
\text{The preheating time:} & \quad t_p = 2*n*t₀ - t_s = 2*10*45 - 10 = 890s (14\text{min 50s}) . \\
\text{The reheating time:} & \quad t_r = (n - 1)*t₀ = (10 - 1)*45 = 405s (6\text{min 45s}) . \\
\text{The breaking time:} & \quad t_{bp} = t_{p,max} - t_p = 25*60 - 890 = 610s , \\
& \quad t_{br} = t_{r,max} - t_r = 12*60 - 405 = 315s , \\
& \quad t_{b,max} = \min(t_{p,max} - t_p, t_{r,max} - t_r) = 315s (5\text{min 15s}) . \\
\text{The time to finish k billets:} & \quad t(k) = t₀*(3*n + k - 2) + (t_r + t_i) = 1295 + 45 * k(s) .
\end{align*}
\]

Substituting the total number of billets N into k, the total operation time is:

\[
\begin{align*}
t_a &= t(N) = t₀*(3*n + N-2) + (t_r + t_i) \\
&= 1290 + 45*N \\
&= 1290 + 45*360 \\
&= 17495s (4h 51\text{min 35s}) .
\end{align*}
\]

### 4.2 Two Operator Model

The entire forging station consists of a forging station and a squashing and trimming station. (See Figure 1) Since one operator cannot operate both the forging and the squashing stations simultaneously, there always exists one idle station. To maximize the usage of equipments, it is necessary to have two operators. The squashing process and the trimming process are done at one station while the forging process is done at the other station. So squashing and trimming processes cannot operate simultaneously. During the time interval tᵣ, one operator forges a billet in tᵣ time and the other operator squashes one billet and trims another billet in (tᵣ+tᵢ) time. The time interval, tᵣ, can be minimized if one station is working all the time. Then t₀ is the maximum of tᵣ and (tᵣ+tᵢ), i.e.,

\[
t₀ = \max(tᵣ, tᵣ + tᵢ) .
\]
The following equations check the temperature history requirement for this model:

\[ t_p = 2^* (n - 1)^* (t_o + t_i); \quad t_{p,\text{min}} \leq t_p \leq t_{p,\text{max}} , \]

\[ t_r = n^* t_o - (t_s + t_i); \quad t_{r,\text{min}} \leq t_r \leq t_{r,\text{max}} . \]

And then we get the breaking time range \( t_{b,\text{max}} \):

\[ t_{bp} = t_{p,\text{max}} - t_p , \]
\[ t_{br} = t_{r,\text{max}} - t_r , \]
\[ t_{b,\text{max}} = \min(t_{bp}, t_{br}) . \]

The time to finish the \( k \) billets is

\[ t(k) = t_o^* (3^* n + (k - 2)) + (t_i + t_i) . \]

Substituting the total number of billets \( N \) into \( k \), the total operation time is

\[ t_s = t(N) = t_o^* (3^* n + (N - 2)) + (t_i + t_i) . \]

**Application:** Again, we applied the current data into this model to get

\[ t_o = \max(t_r, t_s + t_i) \]

\[ = \max(25, 10 + 10) \]

\[ = 25s , \]

\[ 12*60 \geq (n-1) * t_o \geq 6*60 , \]
\[ 25*60 \geq (2*(n-1))*t_o \geq 12*60 , \]
\[ 29 \geq n \geq 16 . \]

For this model, 16 is the best choice. Then the number of raw billets in the furnaces will be 32 \( (N_p = 2^*n) \) and the number squashed billets will be 16 \( (N_r = n) \).

Thus, all other parameters could be calculated as follows:

**The number of furnaces:**

\[ M = 2^* n / m = 2^*16 / 4 = 8 . \]

**The preheating time:**

\[ t_p = 2^* (n - 1)^* t_o + t_i = (2^*16 - 1)^* 25 + 10 = 810s (13min 30s) . \]
The reheating time: 
\[ t_r = n \cdot t_o - (t_s + t_t) = 16 \cdot 25 - (10 + 10) = 380 \text{ s} (6 \text{ min} 20 \text{ s}) . \]

The breaking time: 
\[ t_{bp} = t_{p,max} - t_p = 25 \cdot 60 - 810 = 610 \text{ s} , \]
\[ t_{br} = t_{t,max} - t_t = 12 \cdot 60 - 380 = 340 \text{ s} , \]
\[ t_{t,max} = \min(t_{bp}, \ t_{br}) = 340 \text{ s} (5 \text{ min} 40 \text{ s}) . \]

The time to finish k billets:  
\[ T(k) = t_o \cdot (3 \cdot n + (k - 2)) + (t_t + t_t) = 1185 + 25 \cdot k \text{ s} . \]

Substituting the total number of billets N into k, the total operation time is
\[ t_o = t(N) = t_o \cdot (3 \cdot n + (N - 2)) + (t_t + t_t) \]
\[ = 1185 + 25 \cdot N \]
\[ = 1185 + 25 \cdot 360 \]
\[ = 10185 \text{ s} (2 \text{ h} 49 \text{ min} 45 \text{ s}) . \]

4.3 Time Comparison
As a conclusion, a time comparison table would be a good approach to show the large improvements we achieved with our two models. Based on all the possible relationships between the forging time \( t_f \) and the sum of squashing time and trimming time \( t_s + t_t \), the comparison was developed under three different conditions as described in the Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Current Model</th>
<th>One Operator Model</th>
<th>Two Operator Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operators</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( t_s = 10 \text{ s}, \ t_t = 25 \text{ s}, t_i = 10 \text{ s} )</td>
<td>( t_o )</td>
<td>(1)</td>
<td>45s</td>
</tr>
<tr>
<td>Furnaces (M)</td>
<td>4</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>( n )</td>
<td>36</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>( t_p )</td>
<td>(1)</td>
<td>890s</td>
<td>785s</td>
</tr>
<tr>
<td>( t_t )</td>
<td>(1)</td>
<td>405s</td>
<td>380s</td>
</tr>
<tr>
<td>Total time ( t_s )</td>
<td>22660s ((6h17min40s))</td>
<td>17495s ((4h51min35s))</td>
<td>10185s ((2h49min45s))</td>
</tr>
<tr>
<td>Breaking time</td>
<td>(2)</td>
<td>315s</td>
<td>340s</td>
</tr>
<tr>
<td>( t_s = 10 \text{ s}, \ t_t = 20 \text{ s}, t_i = 15 \text{ s} )</td>
<td>( t_o )</td>
<td>(1)</td>
<td>45s</td>
</tr>
<tr>
<td>Furnaces (M)</td>
<td>4</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>( n )</td>
<td>36</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>( t_p )</td>
<td>(1)</td>
<td>890s</td>
<td>790s</td>
</tr>
<tr>
<td>( t_t )</td>
<td>(1)</td>
<td>405s</td>
<td>375s</td>
</tr>
<tr>
<td>Total time ( t_s )</td>
<td>21040s ((5h50min40s))</td>
<td>17495s ((4h51min35s))</td>
<td>10185s ((2h49min45s))</td>
</tr>
<tr>
<td>Breaking time</td>
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<td>345s</td>
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<tr>
<td>( t_s = 10 \text{ s}, \ t_t = 25 \text{ s}, t_i = 15 \text{ s} )</td>
<td>( t_o )</td>
<td>(1)</td>
<td>50s</td>
</tr>
<tr>
<td>Furnaces (M)</td>
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<td>8</td>
</tr>
<tr>
<td>( n )</td>
<td>36</td>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>
Table 1. Time comparison

* Breaking time – The time operator can use either to fix the break downs of the forging station or to have a rest without the risk of billet-overheating.

(1) No fixed operation time, preheating time and reheating time.
(2) Since the current model does not satisfy the reheating part of temperature history requirement, there is no rest time either.

Through the table, in the best situation, the One Operator Model could save 25% of the total time spent in the current model while keeping the same physical conditions. More surprisingly, in almost all cases, the Two Operator Model could even achieve half of the original total time. Although the extra operator might raise the total labor cost, this will be easily compensated by the cost saved by the maximization of the equipment usage and the large cut in the overall operation time.

4.4 Programming Description
To simulate our models, a computer program was designed. It consists of two major functions. First, it can animate the achieved steady state forging process. Second, by adjusting the input parameters for different products, it can give the corresponding outputs as an operation recommendation. The user-interface (Figure 5) can be mainly divided into three parts: Inputs, Outputs and Simulation.
1. Inputs. As analyzed in the Analysis and Model Development sections, three types of parameters are required as inputs before the execution of the animation: time parameters $t_s$, $t_f$, $t_{p,min}$, $t_{p,max}$, $t_{r,min}$, $t_{r,max}$, the number of billets in one furnace $m$ and the number of billets in one batch $N$. For different products, the time parameters may vary. Depending on the size of the furnace and the requirement of the specific temperature history, the operator can choose an appropriate number of billets to place in each furnace. The number of billets in one batch is also needed to complete all of the input requirements.

2. Output. Based on the input parameters, this program is able to produce the necessary information to run the forging process in the most efficient manner. It gives all of the outputs required to realize the desired steady state condition, using the underlying functions obtained in the model development section. The outputs are: the time length for each step $t_o$, the total time required for finishing the batch $t_a$, the spare time range for a possible machine-breakdown $t_b$, the number of furnaces $M$, and the number of billets in one processing cycle $n$. These outputs can be considered as the recommendation to the operator.

3. Simulation. Once we have inserted the required inputs and hence obtained the resulting outputs, the program shifts to a corresponding mode (which displays the placement of the furnaces) and is ready for the simulation. The movie player interface makes it easy for the user to control the animation process. The different color of each billet indicates the duration of time that it has stayed in the furnace. The deeper the color, the longer it has stayed in the furnace. In addition to the simulation window, there is
also a progress bar which indicates the processing state, and a timer which indicates the time remaining. The task-state window dynamically shows the number of billets that have been produced, are in process (Ph: Being Preheated; Sd: Squashed) and still waiting to be processed (Remained).

5. Recommendations
We have developed two recommendations for Jet Engineering.

The One Operator Model satisfies Jet Engineering’s desire to maintain a single-operator forging process. In this model, the operator opens one furnace to insert a squashed billet, another furnace to insert a cool billet and removes a squashed billet for forging. It is inconvenient to open two different furnaces in one step. However, a special arrangement of the furnaces, such as placing the two open furnaces in closer proximity to one another, can reduce this inconvenience.

The second fact involves Two Operator Model (which Jet Engineering has indicated they were willing to experiment with). This model will theoretically be the most time efficient of the three models. To attain this efficiency, one operator should be skilled in forging while the other experienced at squashing and trimming. It is recommended that the operators be trained to finish their operations at the same time.

6. Future Work
According to the data from Jet Engineering, both of our models will work for all of the current product types. They might not be suitable for some new products in the future unless they avoid following conditions:

\[ t_{p,\text{min}} > 2 t_{r,\text{max}} , \]
\[ t_{p,\text{max}} \leq 2 t_{r,\text{min}} . \]

However, the approach for developing this model is always adaptable. By following our methodology, it will be possible to build other models when necessary.

In our computer program, we employ both the number of billets in one furnace and the temperature history requirement as separate inputs. In fact, the number of billets in the furnace is one of the factors that determine the temperature history requirement. Since the relationship between them has not been discovered and is not part of our assignment, we have not incorporated it into our model. If the relationship can be discovered, the only input will be the number of billets in the furnace while the temperature history requirement will be an output.

The program has included all of the basic information needed for the model we have developed. More information may be added as outputs, depending on the needs of the company.

7. Acknowledgements
The authors acknowledge the Professors Charles R. MacCluer, Ralph Svetic and Eric Torng for their valuable advises and Mr. Timothy Gunn for his help in preparing the project report.

8. References


### 9. Appendices

Appendix A. Table of Processing Time in Current Model

<table>
<thead>
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<th>nth billet</th>
<th>$t_p$</th>
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<th>Total Time in F 1 before Squashed</th>
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<th>Time in F 2 after Squashed</th>
<th>Time in F 2 before Forged</th>
<th>Total Time in F 2 before Forged</th>
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Appendix B. Operation Instructions for One Operator Model

At the beginning, load Furnace 1, 2 and half of 3, from A1 to A10. (Insert one in every 45 seconds.)

Furnace 1  (2)  3  4  (5)(not drawn)

Then, load the rest of Furnace 3, 4 and 5, from B1 to B10. (Insert one every 45 seconds again.)

Furnace 1  (2)  3  4  (5)(not drawn)

Now, the raw billet A1 in Furnace 1 is ready for squashing.

Squash, A1 to A10, and insert cool billets, C1 to C10, to replace them. Insert those squashed billets, As1 to As10, into Furnace 3, 4 and 5, for reheating. Spend 45 seconds on each step, which includes squashing the preheated billet and inserting one cool billet.
The raw billet B1 in Furnace 3 is ready for squashing and the squashed billet As1 in Furnace 3 is ready for forging.

Then the operator begins the first forging cycle.

1. Take out the squashed billet As1 and forge it.

2. Trim the forged one and hence get a final product A1. *(Output)*

3. Take out the raw billet B1 and squash it.

4. Insert the squashed one Bs1 into Furnace 1.
5. Insert a cool billet D1 into Furnace 3. *(Input)*

6. Take out the squashed billet As2 from Furnace 3. Forge it and then trim it. *(Output)*

The last output is in Step 2. From Step3 to 6, the operator squashes one billet, and then forges and trims another billet. The time interval is the sum of these three operation times, which is 45 seconds. Thus, there is one product finished every 45 second.

7. Take out the raw billet B2 from Furnace 3. Squash it and then insert it into Furnace 1 (Bs2).
8. Insert one cool billet D2 into Furnace 3. *(Input)*

The last input is in Step 5. From Step 6 to 8, the operator forges and trims one billet, and then squashes another billet. Again, the time interval is the sum of these three operation times, which is 45 seconds. Thus, there is one billet input every 45 seconds.

After repeating from Step 6 to 8 eight times, the operator finishes the first forging cycle and gets the initial situation back. In this situation, billet groups B, C, and D equal to the billet groups A, B and C in step 1, respectively.
Appendix C. Time Chart for One Operator Model
Appendix D. Operation Instruction for Two Operator Model

At the beginning, load billets, A1 to A16, to Furnaces 1, 2, 3 and 4. (Insert one every 25 seconds.)

Furnace 1 (2) (3) (4) 5 (6) (7) (8) (not drawn)

A1 A2 A3 A4

Then, load billets, B1 to B16, to the Furnaces 5, 6, 7 and 8. (Insert one every 25 seconds again.)

Furnace 1 (2) (3) (4) 5 (6) (7) (8) (not drawn)

A1 A2 A3 A4 B1 B2 B3 B4

Now, the raw billet A1 in Furnace 1 is ready for squashing.

The operators squash the billets in Furnace 1, 2, 3 and 4, from A1 to A16, insert cool billets, Cs1 to Cs16, to replace to them, and insert the squashed billets, As1 to As16, into Furnace 5, 6, 7 and 8, for reheating. Spend 25 seconds on one step, which includes squashing the preheated billet and inserting one cool billet.
The raw billet B1 in Furnace 5 is ready for squashing process and the squashed billet As1 in Furnace 5 is ready for forging process.

Then first forging cycle begins.

1. Operator 1 takes out the squashed billet As1 from Furnace 5 and forges it.

2. Operator 2 takes out the raw billet B1 from Furnace 5 and squashes it. (Operator 1 is forging A1.)

3. Operator 2 inserts the squashed one Bs1 into Furnace 1 for reheating. (Operator 1 is forging A1.)
4. Operator 1 finishes forging A1 and passes it to Operator 2 to trim it.

5. Operator 1 inserts a cool billet D1 into Furnace 5. (INPUT) (Operator 2 is trimming A1.)

6. Operator 1 takes out the squashed billet As2 and forges it. (Operator 2 is still trimming A1.)

7. Operator 2 finishes trimming A1. (OUTPUT) (Operator 1 is doing forging.)
8. Operator 2 takes out the raw billet B2 and squashes it. (Operator 1 is still forging As2.)

<table>
<thead>
<tr>
<th>Furnace 1</th>
<th>(2) (3) (4)</th>
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<th>(6) (7) (8) (not drawn)</th>
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<td>D1 D2 B3 B4</td>
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Till now, the above Step 8 gets back to the same operation as Step 2. Repeat Steps 2~8 to finish all the 16 squashed billets in Furnace 5, 6, 7, and 8.

The final situation in Furnace 1 is same as the initial situation in Furnace 5 and the final situation in Furnace 5 is same as the initial situation in Furnace 1. So the process has returned to its initial state.
Appendix E. Lean Manufacturing Concept

This is a new concept that takes the best features from the American, Japanese and German Industrial traditions and recombines them in a way that can be applied to every economic activity, from long distance travel to construction to health care.

The major tasks of any company are to try to diminish the waste (especially any human activity which absorbs resources but creates no value) and improve production. (And hence a better performance in the market.) Lean concept is a powerful antidote to any waste. It provides a way to specify value, line up value-creating actions in the best possible sequence, conduct these activities without interruption whenever someone requests them, and perform them more and more effectively. In short, Lean thinking is lean because it provides a way to do more and more with less and less – less human effort, less equipment, less time, and less space while coming closer and closer to providing customers with exactly what they want.

Lean thinking also provides a way to make work more satisfying by providing immediate feedback on efforts to convert waste into value. In striking contrast with the recent craze for process reengineering, it provides a way to create new work, rather than simply destroying jobs in the name of efficiency.

As the Lean concept clearly demonstrates, these simple ideas can breathe new life into any company in any industry, routinely doubling both productivity and sales while stabilizing employment. For more details and for a better understanding about The Lean Concept, read the book LEAN THINKING in reference [3].