Investigation of electric field distribution in photonic crystal

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Abstract
We investigate the properties of resonant modes which arise from the introduction of local defects in two-dimensional (2D) photonic crystals. We show that the properties of these modes can be controlled by simply changing the nature and size of the defects. We compute the frequency, polarization, symmetry, and field distribution of the resonant modes by solving Maxwell’s equations in the frequency domain.

KEYWORDS: photonic crystal, electric field, finite difference time-domain

INTRODUCTION
It has been suggested recently that photonic crystals could be used to control the rate of spontaneous emission, since they have the ability of suppressing every mode in the structure for a given range of frequencies.1,2 These crystals behave essentially like three-dimensional dielectric mirrors, reflecting light along every direction in space. In the case where the radiative transition frequency of an atom falls within the frequency gap of the crystal, spontaneous radiative decay can be suppressed. A range of energies which the photonic crystal does not allow photons to propagate, regardless of their direction and polarization is called a complete photonic band gap (PBG) [1].

If a defect is introduced to a perfect photonic crystal, a mode (or group of modes) may be appeared at a frequency (or several frequencies) within the PBG [2]. In this paper, we investigate the properties of these defect states: their frequency, polarization, symmetry, and field distribution, as well as their coupling efficiency to modes outside the crystal. We show that, by choosing a proper defect, we can shape the resonance and tune its frequency to suit most any requirement.
NUMERICAL MODEL AND METHOD

In an isotropic source-free medium, the time-dependent Maxwell’s equations can be written in the following form, [3]:

\[
\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu(r)} \nabla \times \mathbf{E} \quad (1)
\]

\[
\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon(r)} \left[ \nabla \times \mathbf{H} - \sigma(r) \mathbf{E} \right] \quad (2)
\]

where \( \sigma(r) \), \( \mu(r) \), and \( \sigma(r) \) are the position dependent permittivity, permeability, and conductivity of the material, respectively. In a two-dimensional case, the fields can be decoupled into two transversely polarized modes, of [TE] and [TM]. The equations can be discretized in space and time by a so-called Yee-cell algorithm. The following FDTD time stepping formulas are the spatial and time discretizations of Eqs. (1) and (2) on a discrete two-dimensional mesh within the \( x-y \) coordinate system for the TM polarization; while the propagation is in the \( z \)-direction

\[
H_{x_i,j}^{n+\frac{1}{2}} = H_{x_i,j}^{n-\frac{1}{2}} - \frac{\Delta t}{\mu_{n,j}} \frac{\varepsilon_{n,j} E_{y_{i+1,j}} - E_{y_{i,j}}}{\Delta x} \quad (3)
\]

\[
H_{y_i,j}^{n+\frac{1}{2}} = H_{y_i,j}^{n-\frac{1}{2}} + \frac{\Delta t}{\mu_{n,j}} \frac{\varepsilon_{n,j} E_{x_{i+1,j}} - E_{x_{i,j}}}{\Delta y} \quad (4)
\]

\[
\begin{bmatrix}
\Delta t \cdot \frac{E_{n+1,j} - E_{n,j}}{\Delta x} \\
E_{n,j} \\
\Delta t \cdot \frac{E_{n,j+1} - E_{n,j}}{\Delta y}
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_{n,j} - \sigma_{n,j} \frac{\Delta t}{\mu_{n,j}} \\
\varepsilon_{n,j} + \sigma_{n,j} \frac{\Delta t}{\mu_{n,j}}
\end{bmatrix}
\begin{bmatrix}
H_{x_i,j}^{n+\frac{1}{2}} \\
H_{y_i,j}^{n+\frac{1}{2}}
\end{bmatrix}
\]

(5)

where the superscript \( n \) denotes the discrete time step, subscript \( i \) and \( j \) denote the discretized grid point in the \( x-y \) plane, respectively. \( \Delta t \) is the time increment, and \( \Delta x \) and \( \Delta y \) are the intervals between two neighboring grid points along the \( x \) and \( y \) directions, respectively. Similar equations for the TE polarization can be easily obtained.

Special consideration should be given to the boundary of the finite computational domain, where the fields are updated using special boundary conditions, as information out of the domain is not available. Here, we use the perfectly matched layer (PML) for the boundary treatment. In the PML, the electric or magnetic field components are split into two subcomponents (i.e. \( E_x = E_x + E_x \)).

In the TM polarization, losses are assigned to the individual split field components, the effect of which is creating an absorbing medium adjacent to the outer FDTD mesh boundaries. The interface between the PML and the FDTD mesh is reflections for all frequencies, polarizations, and angles of incidence. The FDTD technique can be applied directly for the numerical implementation of the fields inside the PML without any special treatment. If all the necessary information at each grid point, such as the permittivity, permeability, conductivity, and the initial distribution of the fields are known, the time evolution of the fields can be obtained by the discretized FDTD time-stepping formulas. Therefore, this method can be easily applied to any form of inclusions, dielectric and/or metallic. The FDTD time-stepping formulas are numerically stable if the courant’s condition is satisfied [4]:

\[
\Delta t \leq \frac{1}{c \sqrt{\Delta x^2 + \Delta y^2}} \quad (6)
\]

where \( c \) is the speed of the light in vacuum.
Band gap of 2D-PC

The photonic crystal under study consists of a perfect array of infinitely long dielectric rods arranged in a square lattice. Each rod has a radius of 0.20a, where a is the lattice constant and a refractive index of 3.4, which is related to Si (Silicon).

We investigated the propagation of electromagnetic fields in the plane normal to the rods. Since the rods have translational symmetry along their axes, the waves can be decoupled into two transversely polarized modes, of TE and TM. There is a large photonic band gap for the TE polarization between frequencies of 0.29 \((2\pi c/a)\) and 0.42 \((2\pi c/a)\), as depicted in Fig. 1.

(a)

 TE/TM Band Structure

(b)

Fig. 1: Basic properties of a perfect crystal and its band structure; (a): Original lattice, Brillouin zone, lattice constant and material properties. (b): Band diagram for TM and TE polarizations. Band gaps exist only for TE polarization (gray areas).
PC Defect Modes

A defect is now introduced into the perfect array of rods. The defect can possess any shape or size; it can be made by changing the refractive index of a rod, modifying its radius, or removing a rod altogether. The defect could also be made by changing the index or the radius of several rods. Here we modify the radius of a single rod. The modes in the crystal are derived using a super cell approximation, which consists of placing a large crystal with a defect into a super cell and repeating it periodically in space. The considered super cell crystal contains $7 \times 7$ rows. We begin with a perfect crystal, where every rod has a radius of 0.2$a$, and gradually reduce the radius of a single rod. Initially, the perturbation is too small to localize a mode in the crystal. When the radius reaches 0.15$a$, a resonant mode appears in the vicinity of the defect. Since the defect involves removing dielectric material in the crystal, the mode appears at a frequency close to the lower edge of the band gap. As the radius of the rod is further reduced, the frequency of the resonance mode sweeps upward across the gap, and eventually reaches $f=0.38c/a$ when the rod is completely removed. Figure 2 depicts the frequency of the mode for several values of the rod radius. The frequency of the mode can be tuned by simply adjusting the size of the rod.

![Figure 2](image)

In order to couple energy into the cavity, it is necessary to transfer energy through the walls of the crystal. Incident light can transfer energy to the resonant mode by the evanescent field across the array of rods. Plane waves are sent at normal incidence, and the transmission is computed through the crystal. The setup is shown in Fig. 3(a).

The incident light must have some component of the same symmetry as that of the cavity mode in order to couple into the cavity. In the case of a missing rod, we have shown that the resonant mode has even symmetry with respect to the $xz$ plane passing through the middle of the defect. We have also shown that the resonant mode has even symmetry with respect to the $xy$ plane, since the electric field is polarized along the $z$ direction. Therefore, plane waves should be able to couple energy efficiently into the cavity as long as they are polarized along the $z$ direction.
Instead of studying the steady-state response of plane waves, one frequency at a time, a single pulse of light is sent onto the crystal with a wide frequency profile. The spectrum of the incident pulse is shown in Fig. 3(b). It has a Gaussian profile centered at $f = 50.35c/a$ and a width of $0.20c/a$ which extends beyond the edges of the gap. The electric field is polarized along the axis of the rods. The transmission through the crystal is computed at a single point, marked “detector” in Fig. 3(a). The transmission is normalized with respect to the incident amplitude. Results are shown in Fig. 3(c).

A wide gap can be seen in the transmission spectrum. The gap extends from $f = 0.24c/a$ to $f = 0.42c/a$. Although the upper frequency of the gap matches that of Fig. 1, Fig. 3(c) appears to have a larger gap than Fig. 1. We recall, however, that the gap in Fig. 1 applies to all directions in the plane whereas the one in Fig. 3(c) applies only to propagation along the direction of the incident waves. The modes inside the gap are strongly attenuated. They cannot propagate through the crystal and are reflected back.
FIG. 2. Electric-field distribution of TM defect states in an array of dielectric rods for various defect sizes. (a) Monopole, $R = 0.10a$.

(b) and (c) Doubly degenerate dipoles, $R = 0.33a$. (d) and (e) Nondegenerate quadrupoles, $R = 0.60a$. (f) Second-order monopole, $R = 0.60a$. (g) and (h) Doubly degenerate hexapoles, $R = 0.60a$. (i) Dodecapole, $R = 1.00a$. The white circles indicate the position of the rods. FIG. 3. (a) Setup for the computation of the coupling efficiency.

(b) Gaussian frequency profile of the incident pulse. (c) Normalized transmission through the cavity as a function of frequency.

On the other hand, modes outside the gap can be transmitted efficiently; some frequencies have a transmission coefficient close to unity. This suggests that the modes undergo little scattering or reflection as they propagate through the crystal. The rapid fluctuations of the transmission at low frequencies are not real features of the system; they arise from the small signal-to-noise ratio at the edges of the Gaussian frequency profile.

Figure 3(c) also shows the presence of a sharp resonance inside the gap. The coupling efficiency from the incident plane waves to the resonant mode is determined by the height of the peak. Since the resonant mode radiates into a wide range of angles, and since the transmission is computed at a single point in space, only a fraction of the transmitted fields is detected. The coupling efficiency is computed to be slightly larger than 50%.

The electric field distribution of the resonance mode is shown in Fig. 4(a) for the specific case which defect rod the radius is to $0.10a$. The electric field is polarized along the axis of the rods and decays rapidly away from the defect. Since the field does not have a node in the azimuthally direction, it is labeled a monopole. Instead of reducing the size of a rod, it would also have been possible to increase its size. Again, starting from a perfect photonic crystal, we gradually increase the radius of a rod. When the radius reaches $0.25a$, one doubly degenerate modes appear at the top of the gap. Since the defect involves adding material, the modes sweep downward across the gap as the radius increases. The modes eventually disappear into the continuum below the gap when the radius becomes larger than $0.40a$ (see Fig 2). The field distribution of the one doubly degenerate mode is shown in Figs 3(b) for the case where $R = 0.33a$. The modes are labeled dipoles since they have two nodes in the plane. By increasing the radius further, a large number of resonant modes can be created in the vicinity of the defect. This is shown again in Fig 2. Several modes appear at the top of the gap: first a quadruple, then another (no degenerate) quadruple, followed by a second-order monopole and two doubly degenerate hexapoles. These modes also sweep downward across the gap as the defect is increased. The modes are shown in Figs 4(c)–4(d) and 4(e) for the case which $R = 0.60a$.  

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Fig. 3: The electric field distributions of the defect modes for a square array of dielectric rods in the air. (a) Monopole, $R=0.10a$ (b) Doubly degenerate dipoles, $R=0.33a$. (c) monopole, $R=0.60a$. (d) Second-order, $R=0.60a$. and (e) Doubly degenerate hexapoles, $R=0.60a$.

**CONCLUSION**

We have shown that photonic crystals can be used for the fabrication of high-$Q$ microcavities. By introducing a defect in a photonic crystal, sharp resonant states can be created in the vicinity of the defect. The properties of these modes frequency, polarization, symmetry, and field distribution can be controlled by changing the nature and the size of the defect.
REFERENCES


